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A METHOD TO PRE-PROCESS MEASUREMENTS FROM SHIPBORNE GRAVIMETRY USING IMU/GPS: A CASE STUDY ALONG WESTERN COASTAL TAIWAN

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ABSTRACT

We develop an approach to pre-process measurements from shipborne gravimetry using both inertial measurement unit (IMU) and global positioning system (GPS), and explore the feasibility of applying tactical IMU to relevant geoscience applications. The shipborne data used in this paper were collected in the offshore area of western Taiwan on 19th May, 2012. The IMU directly measures the sum of the gravity, ship accelerations in vertical and Eötvös effect. In comparison, the GPS provides the ranging measurements as a function of the ship coordinates. Both the GPS-derived acceleration in vertical (gravity) and the direct gravity measurement from the IMU are pre-processed using the Gaussian filter. Different combinations of filter window widths are tested for a correlation analysis of both IMU-measured and GPS-derived accelerations. The correlation is up to 88.5% when the window width is set to 4.8 s for IMU data and 1 s for GPS data. The amplitudes of the IMU/GPS-derived gravity anomalies are approximately 40 ~ 50 and 20 ~ 30 mgal using the filter window widths of 200 ~ 500 s and 1000 ~ 1500 s, respectively. Overall, this study serves as an important reference for pre-processing the data from shipborne gravimetry using the IMU/GPS.

發展預處理船載 IMU/GPS 重力值觀測量之方法：以台灣西部海岸為例

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關鍵詞：慣性測量元件、全球定位系統、高斯濾波、船載重力測量。

摘　要

本研究發展一方法來預處理利用慣性測量元件 (IMU) 與全球定位系統 (GPS) 所獲得之船載重力值。本文亦研究戰術型 IMU 於地球科學相關應用之可行性。研究中使用的船載資料於 2012 年 5 月 19 日在台灣西部海域所收集。觀測時，IMU 提供重力、船體加速度與科氏力效應之和，而 GPS 則提供距離測量值以計算船體之坐標。研究中無論從 IMU 或 GPS 計算得到的觀測值均使用高斯濾波進行資料預處理，並針對濾波不同罩窗寬度組合計算以分析 IMU 與 GPS 加速度之相關性。研究顯示，當 IMU 與 GPS 分別使用罩窗寬度 4.8 秒和 1 秒處理資料，兩者相關性高達 88.5%。在重力值濾波方面，當使用 200 ~ 500 秒和 1000 ~ 1500 秒的罩窗寬度時，IMU / GPS 計算的重力異常值幅度分別約為 40 ~ 50 mgal 和 20 ~ 30 mgal。整體而言，本研究提供 IMU/GPS 預處理船載重力測量資料之重要參考。

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1. INTRODUCTION

Taiwan is an island: The East faces Pacific Ocean, the West faces Taiwan Strait, the North faces East China Sea and the South faces South China Sea. Marine gravimetry for Taiwan is crucial for the calculations of geoid modeling, circulation movements and ocean currents. Currently, marine gravity data acquisition is mainly from both satellite gravimetry and shipborne gravimetry. The former mainly targets at a large scale but its accuracy is relatively low ([1]). In comparison, the latter has a relatively high accuracy but the cost of the gravimetry is in high economic and time costs.

Inertial measurement units (IMUs) have been widely used for the airborne gravimetry, especially for complex terrains where gravity values are difficult to collect, *e.g.*, mountains. The IMU mainly measures the sum of gravity, object accelerations in vertical and Eötvös effect. The internal accuracy of IMU/GPS-derived airborne gravity can reach approximately 2 mgal with a spatial resolution of 3 km based on past research. Example studies of airborne gravimetry using IMU/GPS are [2-8]. In recent years, there are a number of studies on the IMU/GPS-derived airborne gravity, but rarely on the IMU/GPS-derived shipborne gravity.

In general, the types of IMUs can be divided into two main grades: navigation grade and tactical grade. The former performs the higher accurate than the latter that deserves a relatively low cost. [8] applied both the tactical-grade and navigation-grade IMUs to the airborne gravimetry. The author demonstrated that the gravity anomaly derived by the navigation-grade IMU was deviated from the airborne gravimeter (a LaCoste-Romberg Air-Sea gravimeter) by approximately 0.5 ~ 3 mgal. In contrast, the result derived by the tactical-grade IMU is relatively far from the gravimeter by approximately 20 mgal.

Motivated by [8], the objective of this study is mainly to propose an approach to pre-process measurements from shipborne gravimetry using the tactical IMU combining with GPS. This is the first time to use the low-cost tactical IMU+GPS for the shipborne gravity survey. In the following development, we introduce the theory of the shipborne gravimetry and the corresponding corrections to the gravity measurements in Section 2. Section 3 presents the both data collection and data process and Section 4 shows the gravity anomalies derived by the tactical IMU. The conclusions based on this study is given in Section 5.

2. GRAVIMETRY USING IMU/GPS

The IMU directly measures the sum of the gravity, ship accelerations in vertical and Eötvös effect and the equation of the gravity measurement is shown below ([5] and [9]):

$$g = f_u + \left(\frac{v_e}{R_n + h} + 2\omega_e \cos\varphi \right) v_e + \frac{v_n^2}{R_m + h} \qquad (1)$$

where g is the gravity. f_u is IMU observation which is an integrated vertical acceleration of both the gravity and the vertical acceleration of the ship. v_e and v_n are the E-W and S-N velocity components of the ship, respectively, h is the ellipsoid height of the ship, the term of $\left((v_e / R_n + h) + 2\omega_e \cos\varphi\right) v_e + (v_n^2 / R_m + h)$ corresponds to a correction to the Eötvös effect. Here R_m and R_n is the radius of the Meridian and Prime Vertical, respectively. φ and λ is the latitude and longitude of the ship, and $\omega_e = 7292115 \times 10^{-11}$(rad/s) is the angular velocity of Earth's rotation.

In the shipborne gravimetry, the so-called Coriolis acceleration caused by both the Earth rotation and the ship velocity affects the gravity measurement from IMU/GPS [10,11]. As such, this Eötvös effect needs to be corrected in the gravity measurement.

On the other hand, the precise positions of the ship are estimated through the kinematic positioning technique. The vertical acceleration of the ship is obtained by taking the second derivative of the ship epoch-wise positions in the vertical direction. With the vertical acceleration derived by GPS and the correction to the Eötvös effect, we can extract the gravity signals from the IMU measurements.

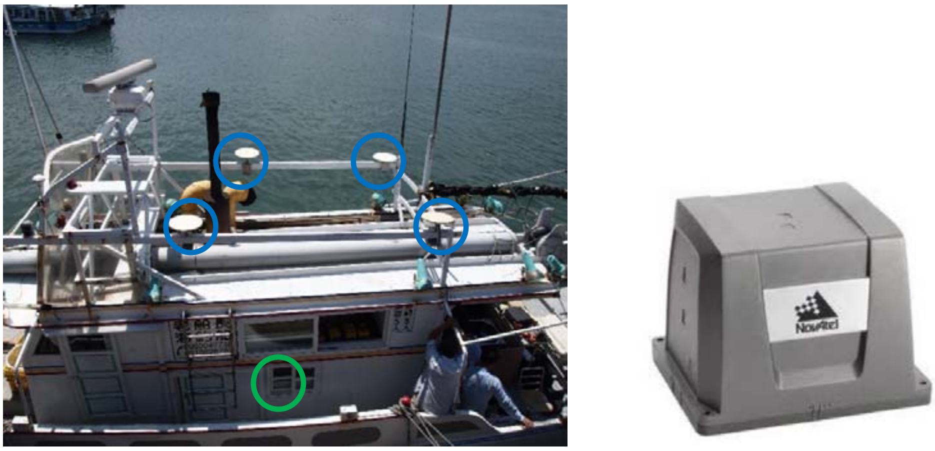
3. SHIPBORNE GRAVIMETRY

3.1 Data acquisition

Figure 1(a) shows the configuration of ship used in this study. The ship is 10.14 m in length and 3.40 m in width, and its total weight is 15.75 tons. Four GPS antennas (blue circle) are mounted on the ship and the IMU is installed at the center of the ship (green circle). The IMU is a tactical-grade IMU (UIMU-LCI) that can obtain 200 Hz measurements, and its shape is shown in Fig. 1(b). The dimension is 168 × 195 × 146 mm and the weight is 4.25 kg. The Accelerometer range and bias are ±16g and < 1.0 mg, respectively. In addition, a geodetic type of GPS receiver, Trimble 5700, is used in collecting 1 Hz GPS signals.

In Fig. 1(a), the four GPS antennas are mainly used for the attitude of the ship. However, for the positioning task, we only select one of the four antennas. Since the distances among these four antennas are very close and the plane formed by the four antennas is approximately horizontal, the vertical positioning result using any of four antennas is assumed to be similar in a case of calm sea waves.

During 14th ~ 20th May 2012, a shipborne survey mission using tactical IMU/GPS was carried out at the offshore region in western Taiwan. In this paper, only the IMU and GPS data collected on 19th May, 2012 are used for the experimental test of shipborne gravity. Figure 2 shows the trajectory of the shipborne gravimetry experiment on 19th May, 2012. Both the IMU and GPS instruments were switched on after the ship sailed 10 km. The total length of the trajectory is approximately 30 km and the total time length of the gravimetry is approximately 20,000 s. After 12,000 s from the start of the gravimetry, the ship stops at the port.



(a) (b)

Fig. 1 (a) The ship configuration. The blue and green circles represent the locations of the GPS antennas and the IMU. (b) The tactical-grade IMU (UIMU-LCI) (source: http://www.forsbergservices.co.uk/uimu-lci-0)

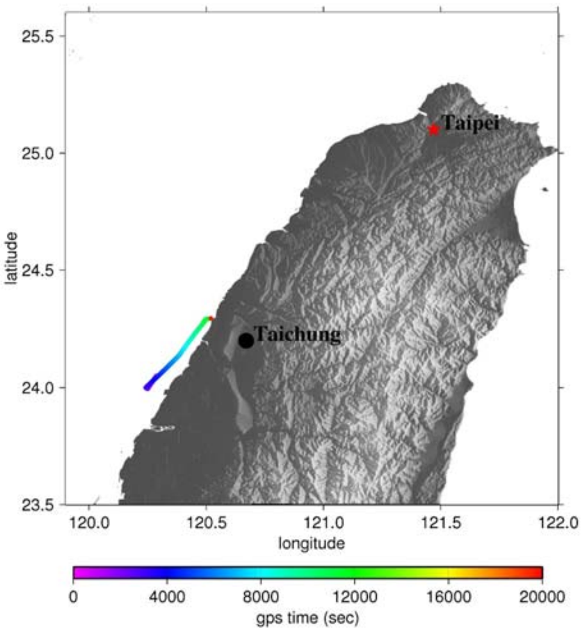


Fig. 2 The trajectory for the shipborne gravimetry. Zero second in the color scale means 00:00:00 (GMT) on 19th May, 2012

3.2 Data Processing

Figure 3 shows the flow chart for processing both the IMU measurements and GPS-derived accelerations. Regarding the GPS data process, we set up a reference station at the port and use the differential kinematic positioning module from the GPS GAMIT/GLOBK package [12]. In this module, the clocks from both the satellite and the receiver are cancelled out. Since the baseline is only 30 km, the positioning accuracy can be approximately determined to 2.1 cm on average [13].

Because the measurement sampling rate of both the GPS and IMU are inconsistent, the IMU data needs to be resampled in consistency with the GPS data. By doing so, (1) the spatial resolution from GPS measurements is similar to that from the IMU ones and (2) the GPS-derived vertical acceleration can correctly be used to remove the quantity of the ship vertical acceleration in f_u. In addition, prior to correct the GPS-derived vertical acceleration to the IMU measurement in Eq. (1), the measurement noise such as ship vibrations related to the attitude-induced noise needs to be minimized or removed. Both the GPS-derived and IMU-measured accelerations show different levels of noise, so we propose different filter strategies to deal with such inconsistent noise.

We developed a code, termed filter_hybrid, mainly to determine an appropriate filter with an optimal windows width (see Appendix A). Users can arbitrarily define the upper and lower limits of the filter window width in the code. In addition, the IMU measurement needs to be correlated with the acceleration derived by GPS in order to define an optimal filter window that is linked to the spatial resolution. This is because most signals in f_u of Eq. (1) results from the vibration of the ship in vertical and can be corrected by using the GPS-derived acceleration.

Subsequently, based on the highest correlation coefficient value, the code is capable of estimating the different filter window widths (up to 0.1 s resolution) for the time series of both IMU measurements and GPS-derived accelerations. The detailed user manual for the filter_hybrid usage can be found in Appendix A.

With the optimal window widths for IMU and GPS, we develop another code, termed IMU_g with the Gaussian filter applied, for the calculation of the gravity. We correct both the GPS-derived accelerations in vertical and the Eötvös effect to the IMU measurements in order to measured gravity values. Finally, the gravity anomaly is obtained by taking the difference between the measured gravity and the normal gravity. A detailed description for the use of IMU_g also can be found in Appendix A. In the following, we discuss the result obtained through the procedure of Fig. 3.

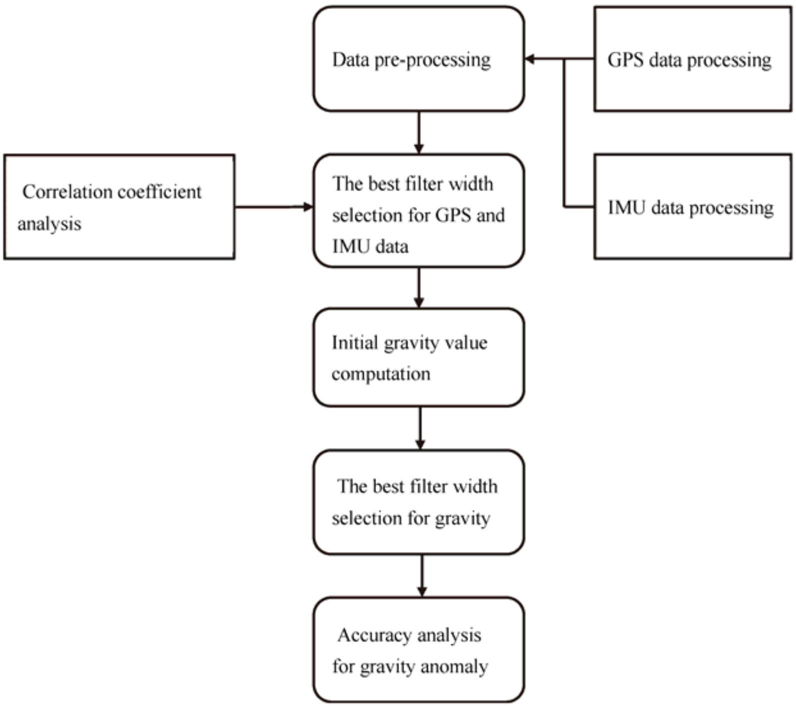


Fig. 3 The flow chart of processing both IMU measurements and GPS-derived accelerations

4. RESULTS FROM PROGRAMS OF FILTER_HYBRID AND IMU_G

4.1 Filtering Result

The acceleration measured by the IMU is the sum of the vertical acceleration of the ship, the true gravity, and the Eötvös effect. Of these, the vertical acceleration, which is highly correlated with the GPS-derived acceleration, is a dominant contributor, while the contributions from both the true gravity and the Eötvös effect are much smaller. Thus, theoretically, the measurement from the IMU should be highly correlated with the vertical acceleration measured by the GPS, and this correlation can be enhanced after filtering.

Figure 4 shows the correlation coefficients between the GPS-derived accelerations in vertical and IMU measurements with different window widths. The range of x-axis from 1 ~ 6s denotes the filter window width for the GPS-derived accelerations, while the range of y-axis from 1 ~ 7s denotes the filter window width for the IMU measurements. From Fig. 4, we can see that the maximum correlation coefficient between GPS-derived accelerations and IMU measurements is 0.885 when adopting window width values of 1 s and 4.8 s for the GPS and IMU data, respectively. With the optimal window widths defined, the corresponding corrections, including the GPS-derived accelerations and Eötvös effect, are applied to Eq. (1) in order to obtain the measurements from the shipborne gravimetry along the trajectiry.

Figure 5 shows the vertical acceleration values derived by the IMU in blue and GPS in red. Figure 5(a) represents the IMU data without any filter processing, and Figs. 5(b) to 5(f) represent the IMU vertical acceleration data after Gaussian filtering by using window widths of 1s, 3 s, 4.8 s, 7 s, and 9 s,

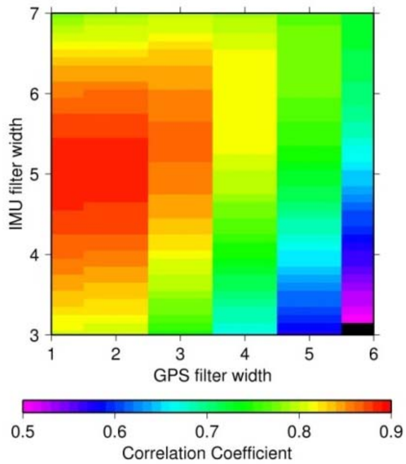


Fig. 4 The correlation coefficients between the GPS-derived and IMU-measured accelerations for different combinations of the window widths

respectively. Because the ship returned to the port at 12,000s (recorded by the GPS) and remained stationary afterwards, both the GPS and IMU accelerations correspondingly decreased. In Fig. 5(a), the GPS acceleration originally varies between approximately ±20,000 mgal, while the IMU acceleration is between ±100,000 mgal. This implies that the noise levels of the two data sets are quite inconsistent. The GPS and IMU data need to be filtered by using different window widths to extract the consistent spatial resolutions. Figures 5(b) to 5(f) present that the IMU vertical acceleration significantly decreases with the increasing filter width. When the filter width is 4.8 s (Fig. 5(d)), the accelerations from IMU are consistent with those derived by GPS.

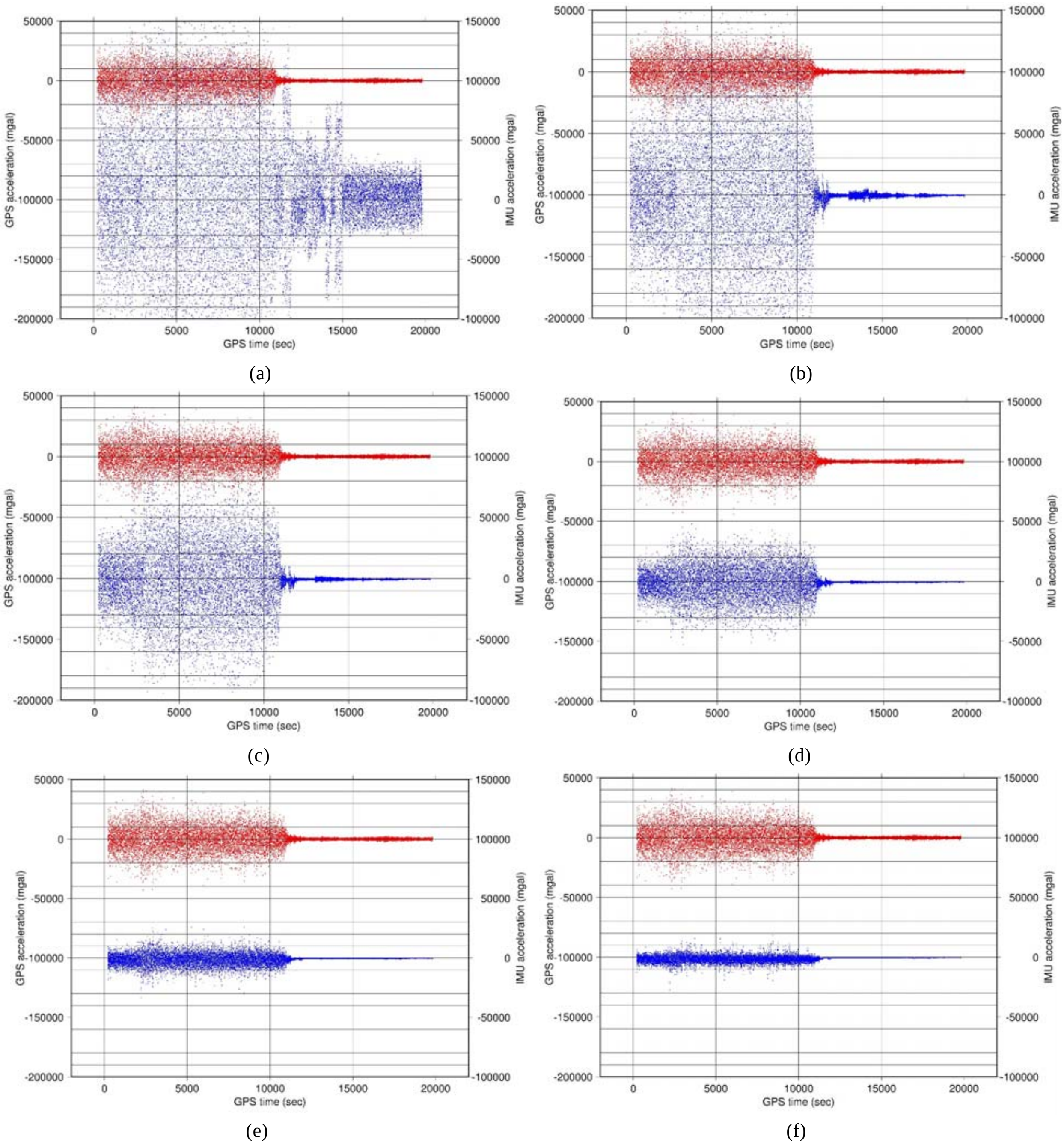


Fig. 5 The vertical accelerations derived from the IMU (in blue) and GPS (in red): (a) represents the IMU-measured acceleration without any filter processing, and (b)-(f) represent the IMU-measured acceleration in vertical using different window widths of 1 s, 3 s, 4.8 s, 7 s, and 9 s, respectively

4.2 Gravity Results

The gravity measurement from IMU is obtain by subtracting both the GPS-derived acceleration in vertical and Eötvös effect. Thus, the gravity anomaly is obtained from the difference between the remaining gravity measurement and the normal gravity which varies with the local latitude. Since the remaining gravity measurement still contains the high-frequency noise, we use the program of IMU_g to remove it with different filter window width. In addition, the gravity anomaly model EGM08 [14,15] is here used as a reference to assess the accuracy of the derived gravity anomaly from the tactical IMU.

Figure 6 shows the filtered gravity anomalies along the trajectory using Gaussian filter with window widths of 200 s, 300 s, 400 s, 500 s, 1000 s, and 1500 s. These gravity anomalies is obtained during the period of the shipborne gravimetry, excluding the period of the ship parking at the port. In Fig. 6, the x-axis represents the GPS time of the day with a unit of seconds, and the y-axis represents the gravity anomalies. The gravity anomalies derived by the tactical IMU are in red and those from EGM08 are in blue. The gravity anomaly

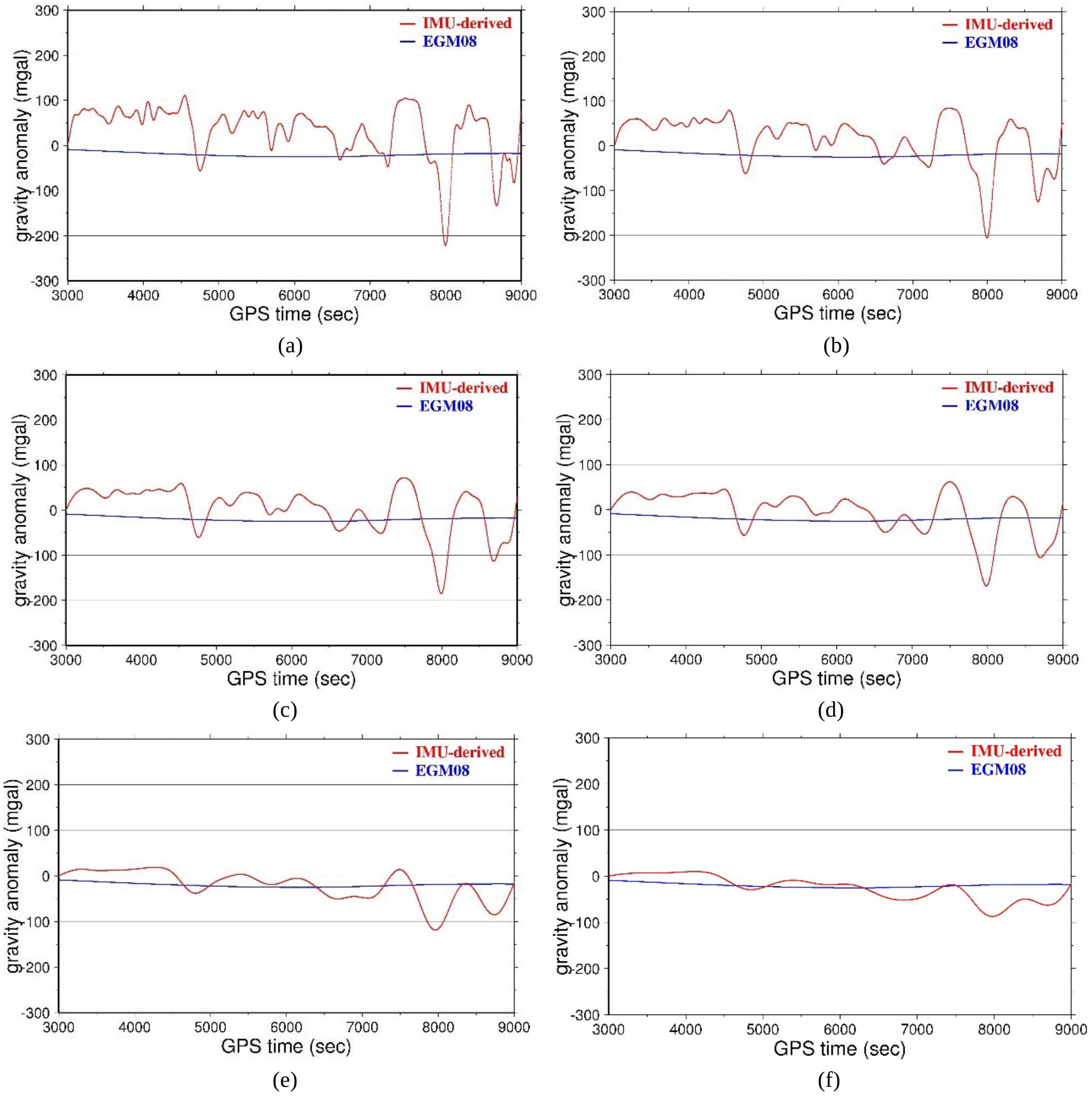


Fig. 6 The filtered gravity anomalies along the ship trajectory using window widths of (a) 200 s, (b) 300 s, (c) 400 s, (d) 500 s, (e) 1000 s, and (f) 1500 s. The gravity anomalies derived by the tactical IMU are in red and those from EGM08 are in blue

Table 1 The difference (unit: mgal) between the EGM08 and the tactical IMU-derived gravity anomalies using Gaussian filter with various window widths

Window width (sec)	Max	Min	Mean	STD
200	130.8	– 202.6	53.3	53.2
300	105.1	– 186.2	34.3	49.2
400	92.4	– 166.3	22.9	46.0
500	187.7	– 82.1	29.3	43.1
1000	39.1	– 99.7	– 1.3	30.8
1500	27.5	– 68.5	– 5.9	23.9

from EGM08 is obtained by interpolating coordinates along the ship trajectory.

In Fig. 6, the gravity anomalies from EGM08 are uniformly distributed over this region and the averaged gravity anomaly along the ship trajectory is approximately –19.9 mgal. The EGM08 is defined on a 5 arc-minute equiangular grid. Due to the spatial resolution, the EGM08 may not be very representative for the accuracy assessment in this case; however, it could be regarded as a valuable reference. Table 1 summarizes the difference between the tactical IMU-derived and EGM08 gravity anomalies. From Fig. 6 and Table 1, the filtered gravity anomaly ranges from –100 to +100 mgal when the window width of 200s is applied, and the corresponding standard deviation (STD) reaches 53.2 mgal. The STD of the gravity anomaly gradually decreases with the increasing filter width. When the window width is set at 1500s that the corresponding spatial resolution is 3 km, the STD is reduced to approximately 20 mgal. According to [8], the accuracy of the gravity anomaly using the tactical IMU on the airborne gravimetry is also close to the 20 mgal when the window width is set at 240 s (about 30 km spatial resolution). As a result, the accuracy of the gravity anomaly derived by the tactical IMU in the shipborne gravimetry is consistent with that in the airborne gravimetry of [8], based on different window widths.

5. CONCLUSIONS

In this work, this is the first time to use the tactical IMU together with GPS for the shipborne gravimetry. The accuracy of the gravity anomaly from this type of IMU is consistent with that given by [8]. In addition, we developed two programs, namely filter_hybrid and IMU_g, for processing the accelerations from both IMU and GPS in extracting the gravity anomaly. The conclusions and suggestions for future studies are listed as follows:

(1) When applying the window width of 1 s and 4.8 s for the GPS-derived and IMU accelerations, the maximum correlation coefficient is 0.885;

(2) When applying the window width of 200 s and 500 s, the accuracy of the gravity anomaly is about 40 ~ 50 mgal. In comparison, when the window width is set up a range from 1000 to 1500s, the accuracy of the gravity anomaly reaches 20 ~ 30 mgal.

To summarize, the tactical IMU is relatively low-cost as compared to the navigation one and this is the first time to utilize the tactical IMU to conduct marine gravimetry in Taiwan. This result shows that the accuracy of the gravity anomaly derived from the shipborne gravitmetry using the tactical IMU could be similar to that derived from the airborne gravimetry. However, the current accuracy of 20 mgal with the spatial resolution of 3 km cannot meet the requirement of geoid determination and relevant geoscience applications. This can be attributed to (1) insufficient measurement accuracy of the tactical IMU and (2) poor GPS-derived positioning accuracy due to the ship vibration and ocean conditions. Based on these two facts, in the future, if higher-level IMUs and large-scale ships can be utilized for shipborne gravimetry, the accuracy of the gravity anomaly can certainly be improved.

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APPENDIX A. SAMPLE BATCH JOBS FOR DATA PROCESSING

(1) The calculation of the optimal filter windows:

filter_hybrid -Agps_input.txt –Bimu_input.txt -Occ.grd3 -Pstd.grd3 -R1/2/5/5.1 -I1/0.1 -M2 -T1

-A：a gps input file; -B: a IMU input file; -O：an output grid recording correlation coefficients; -P: an output grid recording standard deviations; -R: filter window ranges for GPS and IMU data; -I：filter window interval for GPS and IMU data; -M：filter mode selection. 1 for filter only and 2 for iterative filtering by downweighting outliers; -T: filter type selection. 1 for selecting Gaussian filter, 2 for average filter.

(2) The calculation of the shipborne gravity:

IMU_g.f -Agps_input.txt –Bimu_input.txt -I2/5.5/200 –J3000/9000 –Gg_output.txt -M1/1 -T1/1

-A：a gps input file; -B:an IMU input file; -I：filter window widths for GPS data, IMU data, and gravity values; -J： time range (sec) for gravity output ; -G：a gravity output file; -M: filter mode selections for GPS and IMU data, respectively. 1 for filter only and 2 for iterative filtering by downweighting outliers; -T: filter type selections for GPS and IMU data, respectively. 1 for Gaussian filter, 2 for average filter

Both the filter_hybrid and IMU_g programs use the Gaussian filter to process the time series of GPS-derived and IMU-measured accelerations. The Gaussian filter belongs to a low-pass filter. Assuming the point data (x_i, y_i), i = 1, ⋯, n, where x_i is the x-axis, and y_i is the y-axis. The filter equation can be written as follows [16-18]:

$$\hat{y}_k = \left[\sum_{i=i\_\min}^{i=i\_\max} \left(w_{k,i} \times y(i)\right)\right] / \sum_{i=i\_\min}^{i=i\_\max} w_{k,i} \quad (2)$$

Where i_max and i_min are the upper and lower boundaries for the window width defined in the code, σ is 1/6 of the cover window width, $w_{k,i}$ is the weight, and $w_{k,i} = \exp(-(ds/\sigma)^2)$, $ds = x(k) - x(i)$. Figure A1 represents the schematic diagram of the Gaussian filter module.

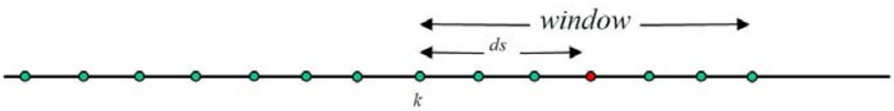

Fig. A1 The schematic diagram of the Gaussian filter module

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